

# The Compressability of Gas

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## Abstract

In many hydraulic systems accumulators are used for various reasons. This ranges from energy storage, dampening of pressure peaks to heave compensation. In all cases a description of the behaviour of the gas is required to determine the size of the volume. This paper explains the well known theories, shows the results and compares these theories. The results show that it is important to check that the molar volume is within the validity of the used theory. Certainly at the high pressure range, the molar volume can get close to or smaller than the critical molar volume, meaning that some theories loose their validity, while others stay more reliable.

Many hydraulic systems use accumulators for reasons as energy storage, dampening of pressure peaks to heave compensation. Especially in the case of heave compensation, it is possible that there are demands from certifying authorities to comply with for instance API Spec 16F or API RP 16Q for drilling applications. These rules demand that the load fluctuation due to the compressing and decompressing of the gas is within a certain bandwidth over the working stroke of the equipment. But also for the other applications there is a need to determine the size of the gas volume [3].

Many theories can be found to describe the gas behaviour. Some are theoretical, like the ideal gas law, others are based on experimental data without any reference to the theory. In this article some of the theories will be shown and a comparison is made between the gas compressibility descriptions.

## 1 Type of gas used

When looking at the hydraulic applications, the used type of gases are air and nitrogen. Both are capable of storing a large amount of energy in the gas, to release when required. This immediately shows the largest threat to the safety aspect: The large amount of energy is easily released, also

when it was not desired. Just for illustration: A water rocket, normally at much lower pressure than an hydraulic system, has achieved altitudes of 830meter, just by the release of this amount of energy. To illustrate: the rocket used Gardena couplings, which go up to 35bar, which is not even close to the working pressure of hydraulic systems. With a simple rocket and a pressure up to 8bar an altitude of approximately 400meter can be achieved with a light water rocket. Another good example is the experiment done by the **program Mythbusters** with the air vessel, where the vessel penetrated a wall. But still, many systems demand a high energy storage, thus it will be used in the industry. Also many codes and standards have rules considering the high pressure, to mitigate the risk. These rules have been properly tested and the industry is accustomed to the rules and regulations, improving the safety of the machinery.

Air is easier to use as nitrogen. Air is freely available, so it will only require a compressor or pressure intensifier unit to get it up to the working pressure of the hydraulic system. The energy storage of air and nitrogen is comparable. Another advantage is that air contains oxygen, so during a leak the oxygen is not pushed away, which is dangerous when people are in the vicinity of the leak. But having oxygen in air is also a large drawback: Due to the oxygen there will

be oxidation of steel inside the system, releasing small particles, which can lead to valve failures over time. Even when the valves are then replaced, the cause of the failures is not solved, so the new valves will also fail due to the same reasons. Another big disadvantage of oxygen in the high pressure gas, is the diesel effect. This effect is caused when the gas is compressed, leading to a temperature increase. When the gas is sufficiently compressed and the temperature is high enough, it can trigger a fire or explosion within the system. Especially at the high pressure at which a hydraulic or heave compensation systems works, the diesel effect will need to be accounted for. It is crucial that all flammable materials, like hydraulic oil (which can pass the hydraulic seals of an piston accumulator) or grease used in the pressure vessels to limit the corrosion are taken into account.

Nitrogen is not directly freely available, but approximately 70% of the air is nitrogen, so it is abundant. Nitrogen generators, which can get the nitrogen out of compressed air, are more and more common in the industry and are improving over time. Nowadays it is possible to have a nitrogen generator installed offshore with sufficient flow, meaning that large storage vessels are no longer required to store all the nitrogen. Important is that the purity is above the 98% to have very little oxidation in the system. With a purity below the 95% there are most certainly problems to be expected considering oxidation. When nitrogen is used which is obtained by liquid nitrogen, the purity goes up to 99.999%. But although the use of nitrogen solves the negative effect of oxygen in the gas, it has the drawback that during a (large) leak it can scatter the oxygen. So when used in confined spaces, the space should have oxygen detectors installed and an alarm should be generated when a too low oxygen level is measured. These rooms should also have sufficient ventilation installed, which is led to a place where it cannot harm anyone. When it is used outside, the wind will normally take care of the ventilation, although it is possible that a cloud of nitrogen is surrounding the leak, especially in wind-still conditions. So personal oxygen meters are required by the personnel, but several suppliers can deliver these personal oxygen meters. These meters also give an alarm when there

is only 19% or less oxygen in the air, to give the employee enough time to escape. For maintenance or commissioning activities, or when safety systems (ventilation) need to be overruled for whatever reason, the employees should also be equipped with escape kits, giving them enough time to escape the room when required.

As both air and nitrogen can be used, the engineer needs to take in account the largest risk: The enormous stored energy. And although air and nitrogen both have their drawbacks, it is to the authors opinion that nitrogen has some slight advantages over air, which is mainly due to the fire and oxidation disadvantages of air.

## 2 Overview of the theory

Several theories can be found, in books as well as on the internet. The theories also have their respective circumstances to where they are valid. In [1] (page 143) there is a good overview of a couple of theories. For application within hydraulic systems, the gas will come close to the critical density ( $\rho < 0.8 \rho_{cr}$ ) at high pressure.

To give a good overview, the first theory will be the ideal gas law. The initial ideal gas law has a constant specific heat ratio  $\kappa$ . As this does not reflect the behaviour at high pressure, it is common in the industry that the specific heat ratio is varied by pressure or by pressure and temperature. Another theory, based on theory as well as experiments, is the compressibility factor. As last theories the experimental based theories are explained, such as the Van der Waals, Beattie-Bridgeman and Benedict-Webb-Rubin. These seven theories show a good overview of what can be used.

### 2.1 Ideal gas law

This law is most often the first law engineers think of when calculating at gas volumes. The ideal gas law is written in an equation:

$$pV = nRT \quad (1)$$

Sometimes the amount of substance  $n$  is replaced by  $n = \frac{m}{M}$ , where  $m$  is the mass in [kg] and  $M$  the molar mass in [kg/mole]. The fraction

$\frac{R}{M}$  is then replaced by the specific gas constant  $R_{specific}$  in  $[\frac{J}{K kg}]$ , leading to:

$$pV = m R_{specific} T \quad (2)$$

The specific gas constant  $R_{specific}$  for air is 286.9 in  $[\frac{J}{K kg}]$ .

For an isothermal process (slow process, gas stays at ambient temperature due to heat exchange with the environment) the equation above can be filled in for the case when only one variable is unknown (for instance the pressure).

When a closed system is assumed (so there is no mass added or extracted from the volume) and the compression or decompression happens quickly, such that no energy (i.e. heat) can exit or enter the system, an adiabatic process is obtained. This requires the first law of thermodynamics:

$$\partial Q = \partial U + \partial W \quad (3)$$

where  $Q$  is heat transfer,  $U$  is the internal energy and  $W$  is the work done by the system (all in  $[J]$ ). As stated, the heat transfer is zero, so  $\partial Q = 0[J]$ . The work done by the system depends on the pressure and the change in volume of the gas ( $\partial W = p\partial V$ ). The internal energy is equal to the change in temperature of the gas ( $\partial U = nC_v\partial T$ ), where  $C_v$  is the specific heat for constant volume (in  $[\frac{J}{mole K}]$ ). When the ideal gas law (equation 1) is combined to rewrite the internal energy to a function of pressure and volume, the more well known equation is found:

$$pV^\kappa = constant \quad (4)$$

where  $\kappa$  is the specific heat ratio, which is the quotient of the specific heat ratios of constant

Explanation of the variables	
$p$	Pressure of the gas in $[Pa]$
$V$	Volume of the gas in $[m^3]$
$n$	Amount of substance of the gas in $[mole]$
$R$	Universal gas constant in $[\frac{J}{K mole}]$ ( $8.314 \frac{J}{K mole}$ )
$T$	Temperature in $[K]$

Table 1: Explanation of the variables of the ideal gas law

pressure and volume ( $\kappa = \frac{C_p}{C_v}$ ). This also results in the following relations (note that the constants of the different equations are not equal to each other):

$$TV^{\kappa-1} = constant \quad (5)$$

$$\frac{T^\kappa}{p^{\kappa-1}} = constant \quad (6)$$

This leads to the following equations, when going from set 1 to set 2:

$$p_1 V_1^\kappa = p_2 V_2^\kappa \quad (7)$$

$$T_1 V_1^{\kappa-1} = T_2 V_2^{\kappa-1} \quad (8)$$

$$\frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^{\frac{\kappa-1}{\kappa}} \quad (9)$$

### 2.1.1 Theoretical ideal gas law

According to the theory, the specific heat ratio is constant, as it is the quotient of the specific heat of constant pressure and volume. For air as well as nitrogen, the specific heat ratio is:

$$\kappa = 1.4 \quad (10)$$

The steps can theoretically be as large as required, as the specific heat ratio is constant. There is thus no maximum step size. With this value the pressure fluctuation to a volume difference can be calculated. Using this value, the description is only valid for low pressure (up to approximately 100bar).

### 2.1.2 Adapted ideal gas law for pressure

As experiments and real life systems show, a real gas often behaves (slightly) different than seen for a constant specific heat ratio. The first step to make this description closer to the reality, a changing specific heat ratio is chosen. For this first step, the specific heat ratio is dependent on pressure, so for changing pressure the specific heat ratio also changes. This means that the formula is fitted to the behaviour of real gas, which extends the range where it is valid. For the example in this article the values as shown in Table 5 are used. The specific heat ratio increases for increasing pressure. Between the values the result is linearly interpolated. As the specific heat ratio changes with pressure, the calculation cannot

take to large steps in pressure, otherwise the approximation will have a too large error, because the specific heat ratio was not changed together with the pressure, which brings the approximation validity again back to the theoretical approach. For this paper the maximum step size in volume is 0.1liter to limit the pressure steps. A small sensitivity analysis shows that making the step size a factor 10 larger or smaller only changes the answer slightly, showing the validity of this step size.

### 2.1.3 Adapted ideal gas law for pressure and temperature

To extend the validity of the theory, the specific heat ratio can be made dependent in pressure as well as temperature. The values for the specific heat ratio for air are shown in Table 6 and for nitrogen are shown in Table 7. Between the values the result is linearly interpolated. This makes the validity larger, especially for larger changes in temperature. Especially for adiabatic processes, the temperature changes along with the pressure, so also for high compression ratios. Also for this approximation the steps need to be small enough, to make sure that the specific heat ratio is changed with pressure and temperature. For this paper the maximum step size in volume is 0.1liter, just as see before. Again, when varying the step size with a factor 10, the result changes only slightly.

## 2.2 Compressibility factor theory

The compressibility factor is much like the ideal gas law, as it only adds a factor to the ideal gas law:

$$Z = \frac{pV}{nRT} \quad (11)$$

The factor Z [-] is now used to describe the real gas behaviour more closely, so it factors the difference of the ideal gas law so to speak. The law can again also be written to the mass form as shown before for the ideal gas law, which is more common in engineering.

$$Z = \frac{pV}{m R_{specific} T} \quad (12)$$

First the mass of the starting conditions is calculated. As the most used systems are closed

systems, there is no mass flow over the borders, thus the mass stays the same for the entire process. As the pressure, volume and temperature are known, it means that the mass is easy to calculate by rewriting equation 12.

For isothermal processes the equation can be solved for pressure, by using numerical force: Start at a low pressure and increase till the calculated pressure and the used pressure are equal, see the appendix for more info.

For an adiabatic process, an estimate for the temperature at the end of the process is required. This is often not known up front, but for this paper, the end temperature of the adapted ideal gas law for pressure and temperature will be used. Otherwise a comparison for adiabatic processes is not possible.

## 2.3 Van der Waals equation

The Van der Waals equation is still based on the ideal gas law, although it has added two experimental determined constants:

$$\left(p + \frac{a}{\nu^2}\right) (\nu - b) = RT \quad (13)$$

Explanation of the variables	
$a$	Molecular interaction coefficient in $[\frac{J m^3}{mole}]$
$b$	Volume of a mole of the gas (size of the molecules) in $[\frac{m^3}{mole}]$
$p$	Pressure of the gas in $[Pa]$
$R$	Universal gas constant in $[\frac{J}{K mole}]$ (8.314 $\frac{J}{K mole}$ )
$T$	Temperature in $[K]$
$\nu$	Molar volume in $[\frac{m^3}{mole}] \frac{V}{n}$

Table 2: Explanation of the variables of the Van der Waals equation

The term  $\frac{a}{\nu^2}$  is to compensate for the intermolecular attraction forces, while the term  $b$  compensates for the volume occupied by the molecules. The molecular interaction coefficient and the volume of a mole of the gas can be cal-

culated as follows:

$$a = \frac{27 R^2 T_{cr}^2}{64 P_{cr}} \quad (14)$$

$$b = \frac{R T_{cr}}{8 P_{cr}} \quad (15)$$

The factors  $P_{cr}$  and  $T_{cr}$  are the critical properties and can be obtained from Table 9. The critical point and its properties are also further explained in Appendix C. The range where the Van der Waals equation is valid is not much wider than for the ideal gas law, although it is historically a step forward to describing a real gas.

For an isothermal process, the temperature does not change, thus when the molar volume is known, the pressure is calculated.

For an adiabatic process however, the temperature needs to be known as well. For this paper this will be estimated using the adapted ideal gas law for pressure and temperature.

## 2.4 Beattie-Bridgeman equation

To gain more validity, the Beattie-Bridgeman equation added more constants to the Van der Waals equation, which resulted in a equation of 5 constants instead of 2. This means that the Beattie-Bridgeman equation extended its validity to 80% of the critical density. The equation is (mind the units as stated in Table 3):

$$p = \frac{RT}{\nu^2} \left(1 - \frac{c}{\nu T^3}\right) (\nu - B) - \frac{A}{\nu^2} \quad (16)$$

Where  $A$  and  $B$  are:

$$A = A_0 \left(1 - \frac{a}{\nu}\right) \quad (17)$$

$$B = B_0 \left(1 - \frac{b}{\nu}\right) \quad (18)$$

The constants are shown in Table 10.

As shown in equation 16, the temperature is required to calculate the pressure. This means that an isothermal process is easy to calculate, but for an adiabatic process the temperature needs to be estimated. As seen before, the temperature of the adapted ideal gas law for pressure and temperature will be used as estimate.

Explanation of the variables	
$p$	Pressure of the gas in [ $kPa$ ]
$R$	Universal gas constant in [ $\frac{J}{K \text{ mole}}$ ] ( $8.314 \frac{J}{K \text{ mole}}$ )
$T$	Temperature in [ $K$ ]
$\nu$	Molar volume in [ $\frac{m^3}{kmole}$ ] $\frac{V}{n}$

Table 3: Explanation of the variables of the Beattie Bridgeman equation

## 2.5 Benedict-Webb-Rubin equation

The last equation is the Benedict Webb Rubin equation, which is an extended Beattie Bridgeman equation and valid up to  $2.5 \rho_{cr}$ . The equation now has 8 constants and is expressed as (mind the units as stated in Table 4):

$$p = \frac{RT}{\nu} + \left(B_0 RT - A_0 - \frac{C_0}{T^2}\right) \frac{1}{\nu^2} + \frac{b RT - a}{\nu^3} \frac{a \alpha}{\nu^6} + \frac{c}{\nu^3 T^2} \left(1 + \frac{\gamma}{\nu^2}\right) e^{-\frac{\gamma}{\nu^2}} \quad (19)$$

Explanation of the variables	
$p$	Pressure of the gas in [ $kPa$ ]
$R$	Universal gas constant in [ $\frac{J}{K \text{ mole}}$ ] ( $8.314 \frac{J}{K \text{ mole}}$ )
$T$	Temperature in [ $K$ ]
$\nu$	Molar volume in [ $\frac{m^3}{kmole}$ ] $\frac{V}{n}$

Table 4: Explanation of the variables of the Benedict Webb Rubin equation

Similar to the Beattie Bridgeman equation, the temperature is required to calculate the pressure. Again, the temperature of the adapted ideal gas law for pressure and temperature is used for the adiabatic process.

Furthermore, there are no specific constants known for air, although there are for nitrogen. This means that the equation is not used for air, but only for nitrogen.

### 3 Comparison of the theories

In this section a comparison is made for adiabatic processes and isothermal processes. Both types of processes are compared on different pressure levels. The results are shown in the appendix in section F.

#### 3.1 Comparison

First the results of an adiabatic process will be discussed. For the very low and low pressure, it is clear that all results are reasonably close to each other. At the very low pressure (5bar), the difference is maximum 5.5%. For the low pressure (50bar) it is maximum of 8.5%. This shows that for this range, the difference in method is not so large. What is curious is that Beattie Bridgeman is at the low end of the spectrum, while Benedict Webb Rubin is at the high end of the spectrum.

For the medium (200bar) and high (300bar) pressure, it becomes obvious that Van der Waals and the theoretical ideal gas law are no longer giving valid answers. The differences between the adapted ideal gas law, Beattie Bridgeman and Benedict Webb Rubin also become significant. The critical volume is shown in table 9. When looking at the molar volume, it is seen that the molar volume is getting close to the critical molar volume. This means that the Beattie Bridgeman is no longer valid for that region. Therefore, only the Benedict Webb Rubin is expected to give the proper answer.

For the isothermal processes, the results are similar as for the adiabatic process. Also the relative difference is comparable with the adiabatic process.

#### 3.2 Influence of the temperature

For the adiabatic process, the temperature at the end of the process is calculated using the adapted ideal gas law for pressure and temperature. This means that this is an estimate, but as the compressibility factor, Beattie Bridgeman and the Benedict Rubens Webb equations are based on measurement data, the temperature is of big influence. When looking at the pressure variation with respect to a small temperature perturbation, it is definitely visible that the temperature

has an influence, dependent on pressure. For the low pressure range, the influence of a couple of Kelvin perturbation, the pressure fluctuates in the range of tenths of a bar. When going to the medium range pressure, the fluctuation is already approximately 1bar to 1.5bar for each Kelvin. This shows that it has an influence, dependent on the pressure, but also that the influence is percentage wise not so large. This verifies that the use of the temperature estimate for these theories is allowable.

### 4 Discussion

For pressures up to medium pressure, the different theories give similar results. When the resulting pressure increases, the differences between the results becomes larger. When looking at the molar volume  $\nu$  for these results, it is seen that these are close to or below the critical molar volume, thus it is exceeding the critical density. This means that in the high range the Beattie Bridgeman is less accurate and the Benedict Webb Rubin is thus more reliable. Looking at the molar volume, it can be concluded that in the high range (above 250bar for 10% volume variation) for hydraulic systems, the Benedict Webb Rubin is more reliable. This means that by choosing the theory to calculate the pressure, the molar volume should be taken into account.

The influence of the temperature for adiabatic processes is present, although at low pressure this is negligible. At higher pressure this becomes more important, but at these molar volumes the chosen theory is of more influence than the temperature. This means that the temperature of the adapted ideal gas law can be used as an estimate, as long as the engineer keeps in mind that it might be slightly different and keeps some safety to cope with this difference.

Just to remind: For hydraulic installations the maximum pressure is normally around 300 to 350bar, as most supplier equipment design pressure is at 300 to 350bar. So only at the maximum operating pressure the difference between the theories becomes significant.

## References

- [1] Yunus A. Çengel and Michael A. Boles. *Thermodynamics, An Engineering Approach*. McGraw Hill, 2008.
- [2] Roland Span et. al. A reference equation of state for the thermodynamic properties of nitrogen from 63.151 to 1000k and pressures to 2200mpa. *AIP Journal of Physics and Chemical Reference Data*, 2000.
- [3] ir. J.G. Gruijters. Hpu design with piston accumulator: An engineering approach. *Not published*, 2014.
- [4] A.D. Kozlov G.A. Spiridonov V.V. Sychev A.A. Vasseerman and V.A. Tsymamy. *Thermodynamic Properties of Air*. Standards, 1978.

# Appendices

## A Ideal gas law

### A.1 Adapted ideal gas law for pressure for air

Below are the used numbers for the ideal gas law adapted for pressure. Linear interpolation is used for results in between the points.

Pressure [bar]	Specific heat ratio $\kappa$
1	1.4
50	1.49
100	1.63
150	1.82
200	2.06
250	2.3
300	2.54
350	2.78
400	3.01
450	3.22
500	3.41

Table 5: Table of pressure and the specific heat ratio for air

### A.2 Adapted ideal gas law for pressure and temperature for air

Below are the used numbers for the ideal gas law adapted for pressure and temperature. These numbers are obtained by measurements of air and found in [4]. Linear interpolation is used for results in between the points.

		Temperature [K]						
		230	270	300	350	400	450	500
Pressure [bar]	1	1.4	1.4	1.4	1.4	1.4	1.39	1.39
	50	1.51	1.49	1.48	1.47	1.46	1.45	1.44
	100	1.72	1.63	1.6	1.56	1.53	1.51	1.49
	150	2.05	1.82	1.74	1.66	1.61	1.58	1.55
	200	2.45	2.06	1.91	1.78	1.7	1.65	1.61
	250	2.86	2.3	2.1	1.91	1.8	1.73	1.68
	300	3.24	2.54	2.28	2.04	1.9	1.81	1.75
	350	3.59	2.78	2.46	2.17	2	1.89	1.81
	400	3.89	3.01	2.64	2.29	2.1	1.97	1.88
	450	4.15	3.22	2.81	2.42	2.19	2.05	1.95
500	4.38	3.41	2.98	2.54	2.29	2.13	2.01	

Table 6: Table of the specific heat ratio as function of temperature and pressure for air

Note that the specific heat ratio which is dependent on pressure only, is the column of 270[K] in this overview.



### A.3 Adapted ideal gas law for pressure and temperature for nitrogen

Below are the used numbers for the ideal gas law adapted for pressure and temperature, but this time for nitrogen. These numbers are obtained by measurements and found in [2]. The article shows the  $c_p$  and  $c_V$  values, and the specific heat ratio is calculated using  $\kappa = c_p/c_V$ . Linear interpolation is used for results in between the points.

		Temperature [K]						
		200	270	300	350	400	450	500
Pressure [bar]	1	1.4	1.4	1.4	1.4	1.4	1.39	1.39
	50	1.68	1.51	1.48	1.46	1.44	1.42	1.41
	100	2.01	1.62	1.56	1.51	1.47	1.45	1.43
	150	2.16	1.70	1.62	1.55	1.50	1.47	1.45
	200	2.14	1.75	1.67	1.58	1.53	1.47	1.45
	250	2.06	1.77	1.69	1.6	1.54	1.5	1.48
	500	1.8	1.72	1.69	1.62	1.57	1.53	1.5

Table 7: Table of the specific heat ratio as function of temperature and pressure for nitrogen

The red coloured ratios are not according the pattern as seen for air: the ratio increases for increasing pressure at the same temperature. This might require more research.

## B Compressibility matrix

The compressibility matrix for air is shown below. For nitrogen the same values are used. The compressibility is basically a factor multiplied with the ideal gas law. This factor is 1 for ideal gas behaviour, but unequal to 1 for real gas behaviour. This way the ideal gas law is used to give a better description of real gas behaviour.

		Pressure [bar]													
		1	5	10	20	40	60	80	100	150	200	250	300	400	500
Temperature [K]	75	0.0052	0.026	0.0519	0.1036	0.2063	0.3082	0.4094	0.5099	0.7581	1.0125				
	80		0.025	0.0499	0.0995	0.1981	0.2958	0.3927	0.4887	0.7258	0.9588	1.1931	1.4139		
	90	0.9764	0.0236	0.0453	0.094	0.1866	0.2781	0.3686	0.4681	0.6779	0.8929	1.1098	1.311	1.7161	2.1105
	100	0.9797	0.8872	0.0453	0.09	0.1782	0.2635	0.3498	0.4337	0.6386	0.8377	1.0395	1.2227	1.5937	1.9536
	120	0.988	0.9373	0.886	0.673	0.1778	0.2557	0.3371	0.4132	0.5964	0.772	0.953	1.1076	1.5091	1.7366
	140	0.9927	0.9614	0.9205	0.8297	0.5856	0.3313	0.3737	0.434	0.5909	0.7699	0.9114	1.0393	1.3202	1.5903
	160	0.9951	0.9748	0.9489	0.8954	0.7803	0.6603	0.5696	0.5489	0.634	0.7564	0.884	1.0105	1.2585	1.497
	180	0.9967	0.9832	0.966	0.9314	0.8625	0.7977	0.7432	0.7084	0.718	0.7986	0.9	1.0068	1.2232	1.4361
	200	0.9978	0.9886	0.9767	0.9539	0.91	0.8701	0.8374	0.8142	0.8061	0.8549	0.9311	1.0185	1.2054	1.3944
	250	0.9992	0.9957	0.9911	0.9822	0.9671	0.9549	0.9463	0.9411	0.945	0.9713	1.0152	1.0702	1.199	1.3392
	300	0.9999	0.9987	0.9974	0.995	0.9917	0.9901	0.9903	0.993	1.0074	1.0326	1.0669	1.1089	1.2073	1.3163
	350	1	1.0002	1.0004	1.0014	1.0038	1.0075	1.0121	1.0183	1.0377	1.0635	1.0947	1.1303	1.2116	1.3015
	400	1.0002	1.0012	1.0025	1.0046	1.01	1.0159	1.0229	1.0312	1.0533	1.0795	1.1087	1.1411	1.2117	1.289
	450	1.0003	1.0016	1.0034	1.0063	1.0133	1.021	1.0287	1.0374	1.0614	1.0913	1.1183	1.1463	1.209	1.2778
	500	1.0003	1.002	1.0034	1.0074	1.0151	1.0234	1.0323	1.041	1.065	1.0913	1.1183	1.1463	1.2051	1.2667
	600	1.0004	1.0022	1.0039	1.0081	1.0164	1.0253	1.034	1.0434	1.0678	1.092	1.1172	1.1427	1.1947	1.2475
800	1.0004	1.002	1.0038	1.0077	1.0157	1.024	1.0321	1.0408	1.0621	1.0844	1.1061	1.1283	1.172	1.215	
1000	1.0004	1.0018	1.0037	1.0068	1.0142	1.0215	1.029	1.0365	1.0556	1.0744	1.0948	1.1131	1.1515	1.1889	

Table 8: Table of the compressibility matrix as function of temperature and pressure

The used Matlab code for determining the end pressure is:

```

mass=p1*V1/(interp2(pk*10^5, Tk, Z, p1, T1)*Rn*T1);
dP = 0.01*10^5;% [Pa] Pressure stepsize
ploop= 1*10^5; % [Pa] Start loop p
pguess= 100*10^5; % [Pa]
5 T2 = T1;% [K] End temperature
while ploop<pguess;
    ploop=ploop+dP;
    pguess=interp2(pk*10^5, Tk, Z, ploop, T2)*mass*Rn*T1/V2;
end
10 p2=pguess*10^-5;

```

## C Van der Waals equation

The two constants of the Van der Waals equation can be calculated using the critical point properties of the gas as shown in Table 9.

The critical point is a thermodynamical phenomenon considering a liquid turning into gas. When looking for instance at water, the properties of the liquid  $H_2O$  and the gas (steam)  $H_2O$  are not so much alike. The liquid water is nearly incompressible for instance, while the gas is compressible. When getting closer to the critical point by increasing pressure, these properties start to change, such that at the critical point there is no real difference anymore between the gaseous form and the liquid form.

	$M$ $\frac{kg}{K\ mole}$	$R_{specific}$ $\frac{J}{kg\ K}$	Critical point properties		
			$T_{cr}$ $K$	$P_{cr}$ $MPa$	$\nu_{cr}$ $\frac{m^3}{kmol}$
Air	28.97	286.9	132.5	3.77	0.0883
$N_2$	28.013	296.8	126.2	3.39	0.0899

Table 9: Table of the critical point properties of air and nitrogen

## D Beattie Bridgeman equation

In the table below the constants used in the Beattie Bridgeman equation are shown.

	$A_0$	$a$	$B_0$	$b$	$c$
Air	131.8441	0.01931	0.04611	-0.001101	$4.34 \cdot 10^4$
$N_2$	136.2315	0.02617	0.05046	-0.00691	$4.20 \cdot 10^4$

Table 10: Table of the constants as used in the Beattie Bridgeman equation

## E Benedict Webb Rubin equation

In the table below the constants used in the Benedict Webb Rubin equation are shown.

	$a$	$A_0$	$b$	$B_0$	$c$	$C_0$	$\alpha$	$\gamma$
$N_2$	2.54	106.73	0.002328	0.4074	$7.379 \cdot 10^4$	$8.164 \cdot 10^5$	$1.272 \cdot 10^{-4}$	0.0053

Table 11: Table of the constants as used in the Benedict Webb Rubin equation

## F Results of adiabatic process

Two types of calculations are done: medium volume variation and high volume variation. The results are shown in the tables below. The pressure is varied from very low pressure (5bar), low pressure (50bar), medium pressure (200bar) to high pressure (300bar). The resulting pressure is shown in the tables.

### F.1 Medium volume variation

For these results the volume is varied from 1000liters to 900liters. The calculation to 1100liters is done as well, but the resulting pressure difference is equal to what is shown here, thus these results are not shown separately, as this would not add anything to the results. The starting temperature is 293K.

		Very low pressure 5bar $T_2 = 305.8439K$ $\nu_2 = 4.3848 \frac{m^3}{kmole}$	Low pressure 50bar $T_2 = 308.5184K$ $\nu_2 = 0.4385 \frac{m^3}{kmole}$	Medium pressure 200bar $T_2 = 324.4327K$ $\nu_2 = 0.1096 \frac{m^3}{kmole}$	High pressure 300bar $T_2 = 337.3555K$ $\nu_2 = 0.0731 \frac{m^3}{kmole}$
Ideal	Theory	5.79bar	57.95bar	231.79bar	347.68bar
	Pressure	5.8bar	58.57bar	251.55bar	401.33bar
Air	Press+Temp	5.8bar	58.5bar	246.06bar	383.79bar
	Compressibility	5.78bar	58.87bar	261.85bar	431.37bar
	V.d. Waals	5.78bar	56.75bar	256.01bar	513.01bar
	Beattie Bridg.	5.5bar	55.46bar	257.49bar	458.07bar
$N_2$	Press.+Temp	5.80bar	58.48bar	238.52bar	358.25bar
	V.d. Waals	5.78bar	57.03bar	266.26bar	559.01bar
	Beattie Bridg.	5.52bar	55.19bar	244.02bar	417.47bar
	Benedict W. R.	5.79bar	58.44bar	277.42bar	520.17bar

Table 12: Table of the results of the adiabatic calculation with medium volume change

### F.2 High volume variation

For these results the volume is varied from 1000liters to 700liters. The calculation to 1300liters is done as well, but the resulting pressure difference is equal to what is shown here, thus these results are not shown separately, as this would not add anything to the results. The starting temperature is 293K.

		Very low pressure 5bar $T_2 = 338.9672K$ $\nu_2 = 3.4104 \frac{m^3}{kmole}$	Low pressure 50bar $T_2 = 351.268K$ $\nu_2 = 0.3410 \frac{m^3}{kmole}$	Medium pressure 200bar $T_2 = 419.9492K$ $\nu_2 = 0.0853 \frac{m^3}{kmole}$	High pressure 300bar $T_2 = -$ $\nu_2 = 0.0568 \frac{m^3}{kmole}$
Ideal	Theory	8.24bar	82.38bar	329.53bar	494.29bar
	Pressure	8.27bar	86.47bar		
Air	Press+Temp	8.26bar	85.63bar	409.51bar	
	Compressibility	8.28bar	88.08bar		
	V.d. Waals	8.24bar	84.23bar	529.61bar	
	Beattie Bridg.	7.85bar	82.99bar	498.79bar	
$N_2$	Press.+Temp	8.26bar	85.13bar	358.51bar	
	V.d. Waals	8.24bar	84.81bar	561.53bar	
	Beattie Bridg.	7.88bar	82.31bar	467.83bar	
	Benedict W. R.	8.27bar	87.34bar	556.2bar	

Table 13: Table of the results of the adiabatic calculation with high volume change

## G Results of isothermal process

Two types of calculations are done: medium volume variation and high volume variation. The results are shown in the tables below. The pressure is varied from very low pressure (5bar), low pressure (50bar), medium pressure (200bar) to high pressure (300bar). The resulting pressure is shown in the tables.

### G.1 Medium volume variation

For these results the volume is varied from 1000liters to 900liters. The starting temperature is 293K.

		Very low pressure 5bar $\nu_2 = 4.3848 \frac{m^3}{kmole}$	Low pressure 50bar $\nu_2 = 0.4385 \frac{m^3}{kmole}$	Medium pressure 200bar $\nu_2 = 0.1096 \frac{m^3}{kmole}$	High pressure 300bar $\nu_2 = 0.0731 \frac{m^3}{kmole}$
	Ideal gas law	5.56bar	55.56bar	222.22bar	333.33bar
Air	Compressibility	5.53bar	55.51bar	226.29bar	348.34bar
	V.d. Waals	5.53bar	53.54bar	220.26bar	412.13bar
	Beattie Bridg.	5.27bar	52.3bar	222.92bar	372bar
$N_2$	V.d. Waals	5.53bar	53.81bar	229.42bar	451.79bar
	Beattie Bridg.	5.29bar	52.06bar	210.35bar	334.4bar
	Benedict W. R.	5.55bar	55.15bar	239.76bar	419.2bar

Table 14: Table of the results of the isothermal calculation with medium volume change

## G.2 High volume variation

For these results the volume is varied from 1000liters to 700liters. The starting temperature is 293K.

		Very low pressure 5bar $\nu_2 = 3.4104 \frac{m^3}{kmole}$	Low pressure 50bar $\nu_2 = 0.3410 \frac{m^3}{kmole}$	Medium pressure 200bar $\nu_2 = 0.0853 \frac{m^3}{kmole}$	High pressure 300bar $\nu_2 = 0.0568 \frac{m^3}{kmole}$
	Ideal gas law	7.14bar	71.43bar	285.71bar	428.57bar
Air	Compressibility	7.11bar	71.27bar	311.07bar	
	V.d. Waals	7.1bar	68.32bar	313.04bar	778.81bar
	Beattie Bridg.	6.77bar	67.27bar	303.03bar	539.31bar
N <sub>2</sub>	V.d. Waals	7.1bar	68.79bar	313.04bar	917.97bar
	Beattie Bridg.	6.79bar	66.72bar	278.29bar	464.44bar
	Benedict W. R.	7.13bar	70.98bar	332.83bar	666.23bar

Table 15: Table of the results of the isothermal calculation with high volume change